

Frequency Modulation (FM)

Modules: Audio Oscillator, Wideband True RMS Meter, VCO, Utilities, Twin Pulse Generator, Tuneable LPF, Multiplier, Adder, Phase Shifter, Digital Utilities, 100-kHz Channel Filters, Laplace Transform, FM Utilities, Speech, Headphones

0 Pre-Laboratory Reading

Frequency modulation (FM) is an analog modulation technique for which the message signal is conveyed through variations in the frequency of the carrier. FM is used for radio broadcasting, public-safety radio (police and fire), marine radio, amateur (ham) radio, and general mobile radio service (walkie-talkie). FM is also used in radar.

0.1 Voltage-Controlled Oscillator (VCO)

A carrier with FM is of the form

$$y(t) = \cos \left[2\pi f_c t + 2\pi \Delta f \int_0^t x_n(t') dt' \right] \quad (1)$$

where f_c is the nominal carrier frequency, Δf is the frequency deviation (a parameter), and $x_n(t)$ is the normalized message signal. The instantaneous (cyclical) frequency of this carrier is found by taking the derivative of the argument of the cosine and then dividing by 2π . According to the Fundamental Theorem of Calculus,

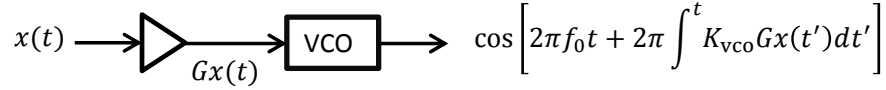
$$\frac{d}{dt} \int_0^t x_n(t') dt' = x_n(t) \quad (2)$$

Therefore,

$$\frac{1}{2\pi} \cdot \frac{d}{dt} \left[2\pi f_c t + 2\pi \Delta f \int_0^t x_n(t') dt' \right] = f_c + \Delta f x_n(t) \quad (3)$$

The instantaneous frequency for the carrier of Eq. (1) is $f_c + \Delta f x_n(t)$. In words, the instantaneous frequency is offset from the nominal carrier frequency by an amount that is proportional to the message signal.

One straightforward method for generating this signal is with a VCO.



The parameter K_{vco} has units of hertz per volt, and it represents the sensitivity of the VCO output frequency to the VCO input voltage. For the VCO used in this experiment, K_{vco} is adjustable but always negative. This means that a positive voltage at the VCO input produces a frequency at the VCO output that is less than the nominal VCO frequency f_0 . In this experiment, an amplifier with negative gain will always be used in front of the VCO so that the two minus signs cancel. A positive value for $x(t)$ will produce a positive offset in the VCO output frequency. When a VCO is used to generate a carrier, the nominal VCO frequency will equal the nominal carrier frequency. That is, $f_c = f_0$.

The maximum of the absolute value of the message signal $x(t)$ is of importance; this is denoted here by p ($p \geq 0$).

$$p = \max |x(t)| \quad (4)$$

The normalized message signal $x_n(t)$ is:

$$x_n(t) = x(t)/p \quad (5)$$

The maximum of the absolute value of $x_n(t)$ is therefore

$$\max |x_n(t)| = 1 \quad (6)$$

The expression for the VCO output equals the right-hand side of Eq. (1) when

$$\Delta f = pGK_{vco} \quad (7)$$

In this expression, p is positive. G and K_{vco} are both negative. The frequency deviation Δf is therefore positive; it represents, for a given message signal $x(t)$, the maximum deviation of the carrier instantaneous frequency from the nominal frequency. With $-p \leq x(t) \leq p$, the instantaneous carrier frequency lies, at every instant in time, between $f_c - \Delta f$ and $f_c + \Delta f$. It should be noted that Δf is in hertz, since p is in volts, G is dimensionless and K_{vco} has units hertz/volt.

0.2 Bandwidth of FM Carrier

It is important to estimate the bandwidth of a carrier with FM. This bandwidth depends, in general, on both the bandwidth W of the message signal and the frequency deviation Δf . One common estimate of the bandwidth is Carson's rule:

$$\text{Bandwidth of FM Carrier} \cong 2(\Delta f + W) \quad (8)$$

In the case where the message signal bandwidth is much larger than the frequency deviation, this bandwidth is approximately $2W$, the same bandwidth as for DSB and AM. However, for most practical FM systems $\Delta f > W$. The bandwidth of the typical FM system exceeds that for DSB and AM.

0.3 Sinusoidal Message and Bessel Functions

As usual, a sinusoidal message is employed in testing. In this case, the normalized message signal is modeled as

$$x_n(t) = \cos(2\pi f_m t) \quad (9)$$

with frequency f_m . The integral within the right-hand side of Eq. (1) becomes

$$\int_0^t x_n(t') dt' = \frac{\sin(2\pi f_m t)}{2\pi f_m} \quad (10)$$

A modulation index β for FM is defined as

$$\beta = \frac{\Delta f}{f_m} \quad (11)$$

β is dimensionless. Combining Eqs. (1), (10) and (11),

$$y(t) = \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad (12)$$

For FM, as with all analog modulation schemes, the spectrum of the modulated carrier consists of a set of spectral lines whenever the message signal is a sinusoid. For FM the spectral lines are located at the frequencies

$$f_c + k f_m, \quad k = 0, \pm 1, \pm 2, \pm 3, \dots$$

The spectral lines corresponding to positive values of k are called *upper sidebands*. The spectral lines corresponding to negative values of k are called *lower sidebands*. The spectral line at frequency f_c (corresponding to $k = 0$) is called the *carrier*. For the sake of clarity, this spectral line is also sometimes called the *residual carrier*.

The power P_k in the spectral line at $f_c + k f_m$ is given by

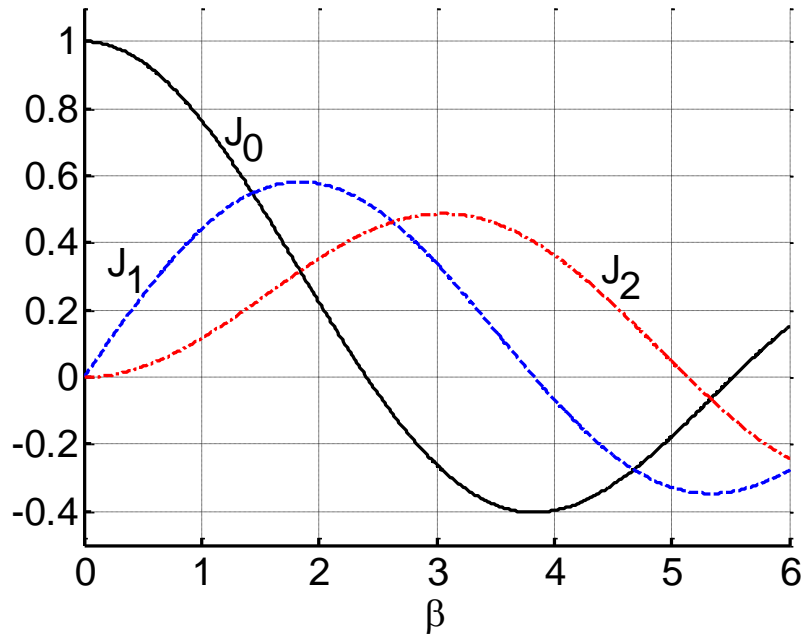
$$P_k = J_k^2(\beta) \cdot P \quad (13)$$

where P is the total power in the modulated carrier and $J_k(\cdot)$ is a Bessel function of the first kind of order k . Eq. (13) is based on a one-sided line spectrum. That is to say, all of the power in the modulated carrier is taken into account within the positive half of the frequency line.

Bessel functions can be easily computed in MATLAB and many other programs for numerical analysis. $J_k(\beta)$ can be computed like this:

`besselj(k, beta)` (MATLAB)

The Bessel functions $J_0(\cdot)$, $J_1(\cdot)$ and $J_2(\cdot)$ are plotted below.



Bessel functions of the first kind of orders 0, 1, and 2

There are several properties of Bessel functions that are useful in the theory of FM. The following property, for integer order k ,

$$J_{-k}^2(\beta) = J_k^2(\beta) \tag{14}$$

means that

$$P_{-k} = P_k \tag{15}$$

In words, two sidebands that are symmetrically located about f_c (a lower sideband and an upper sideband) contain the same power. For example, the lower fundamental sideband, at $f_c - f_m$, and the upper fundamental sideband, at $f_c + f_m$, have the same power. Another property of Bessel functions is

$$\sum_{k=-\infty}^{\infty} J_k^2(\beta) = 1 \quad (16)$$

Combing this property with Eq. (13) gives

$$\sum_{k=-\infty}^{\infty} P_k = P \quad (17)$$

In words, the sum of the power in all sidebands (lower and upper) plus the power in the residual carrier equals the total power of the modulated carrier.

In experimental work it is useful to have expressions for the power in a spectral line relative to that in the residual carrier. In decibels, this is

$$\frac{P_k}{P_0} = 20 \log \left[\frac{J_k(\beta)}{J_0(\beta)} \right] \text{ dBc} \quad (18)$$

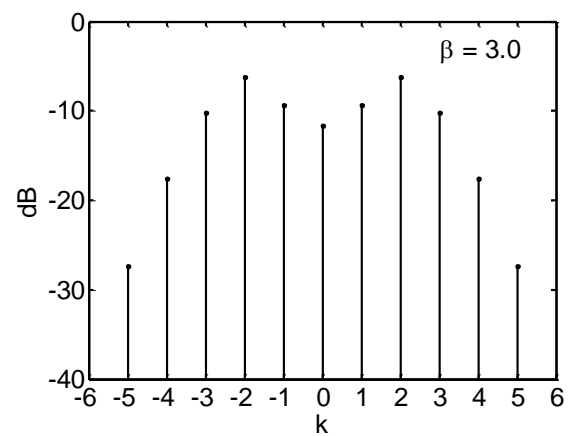
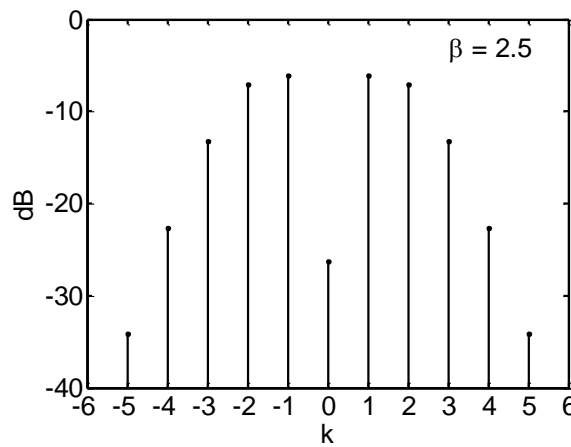
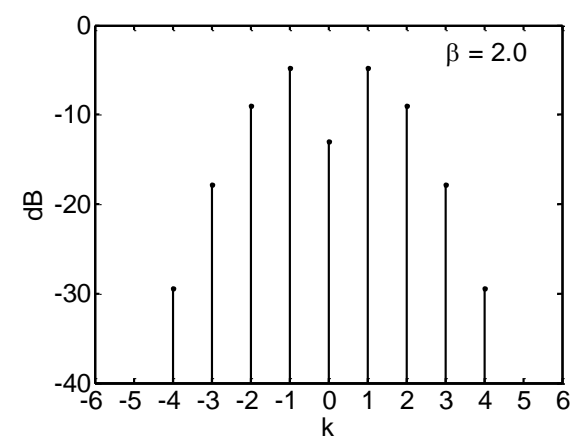
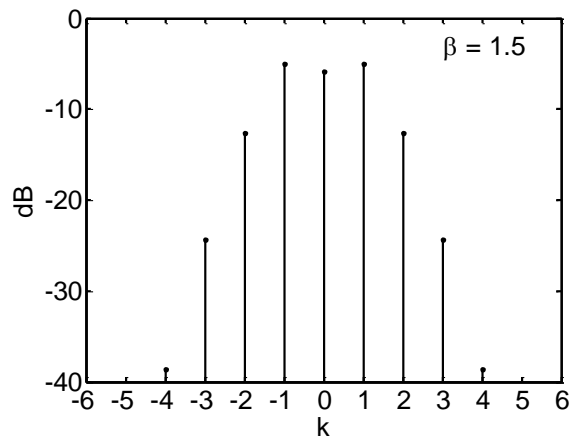
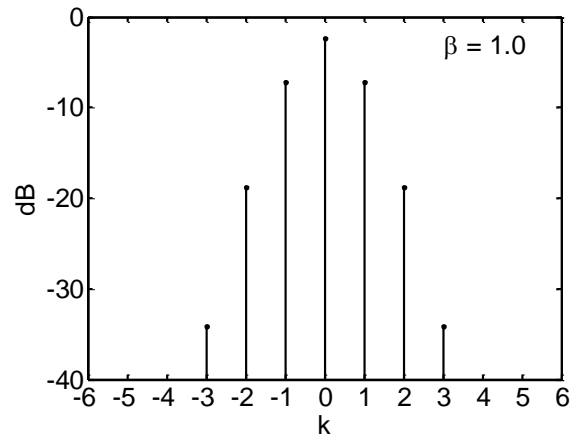
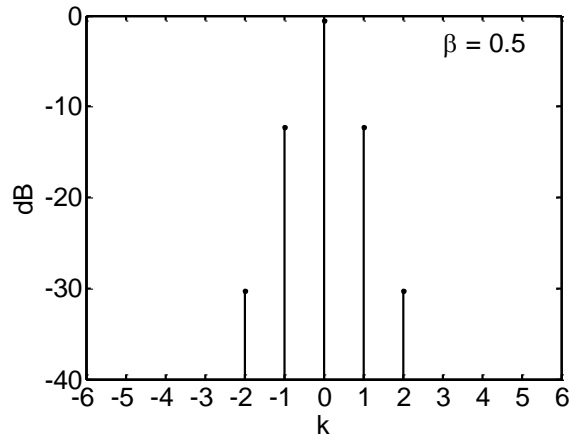
where, as usual, dBc means decibels relative to the (residual) carrier. Table 1 lists P_1/P_0 for several values of β . This table indicates, for example, that when $\beta = 0.5$, the upper fundamental and the lower fundamental will each be smaller than the line at f_c (the residual carrier) by 11.8 dB. As a second example, when $\beta = 2.0$, the upper fundamental and the lower fundamental will each be larger than the residual carrier by 8.2 dB.

Table 1: P_1/P_0 as a Function of β for a Sinusoidal Message

β	P_1/P_0 (dBc)
0.5	-11.8
1.0	-4.8
1.5	0.7
2.0	8.2
2.5	20.2
3.0	2.3

For a sinusoidal message signal, Carson's rule may be modified as follows. The message signal bandwidth W equals f_m in this case. Using Eqs. (8) and (11), the estimate of bandwidth for the modulated carrier becomes

$$\text{Bandwidth of FM Carrier} \cong 2f_m(\beta + 1) \quad (19)$$

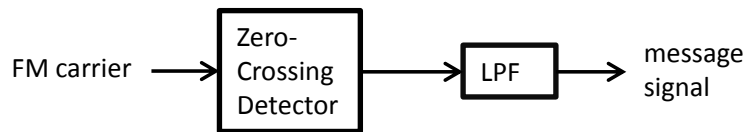


Spectrum of an FM carrier with sinusoidal message signal: P_k/P (dB) versus k

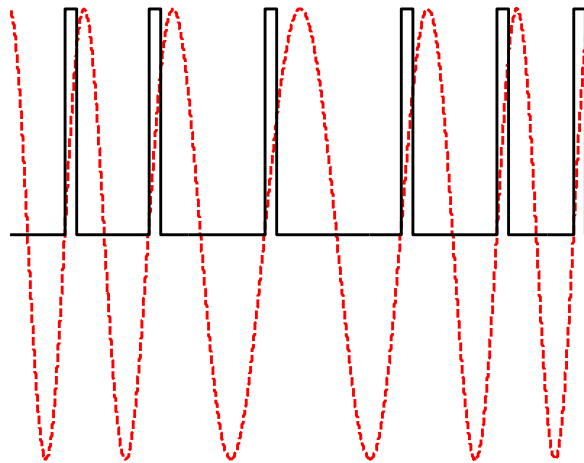
When the message signal is a sinusoid with a given frequency f_m , the number of significant spectral lines increases as the modulation index increases. The bandwidth of the modulated carrier is in approximate agreement with Carson's rule as given in Eq. (19).

0.4 Zero-Crossing Detector

One simple method of demodulating an FM carrier is by means of a zero-crossing detector.



The FM carrier enters a zero-crossing detector. This detector produces a pulse for every positive-going zero-crossing in the signal on its input. All pulses have a common height and a common width. When the instantaneous frequency of the FM carrier is relatively large, the pulses are closely spaced. When the instantaneous frequency is relatively small, the pulses are less frequent.



FM carrier (dashed) and the output of a zero-crossing detector (solid)

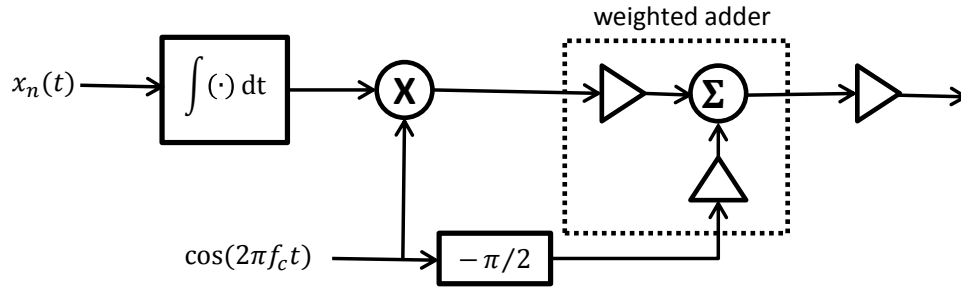
When the output of the zero-crossing detector is passed through a low-pass filter, the result can be a reasonable approximation of the message signal. For this to work, the bandwidth of the low-pass filter must be greater than the bandwidth of the message signal but less than the nominal carrier frequency.

0.5 Armstrong Modulator and Narrowband FM

It was explained above how to generate FM by the direct method (that is, using a VCO). Here an indirect method of generating FM is explained. In this indirect method, the message signal is

integrated; this produces a new signal whose derivative is the original message signal. Phase modulation of this new signal onto the carrier produces the FM of Eq. (1).

Here a special type of phase modulator, called the *Armstrong modulator*, is used. This phase modulator is only an approximation to a true phase modulator; however, it is a good approximation for small modulation indices. The combination of an integrator and an Armstrong modulator is illustrated below; this represents an approximation to a frequency modulator. The weighting factors in the weighted adder are shown explicitly as amplifiers. In the TIMS weighted adder, the gains of both of these amplifiers are negative. This can be neutralized by placing an amplifier with negative gain at the output of the weighted adder.



The output of the integrator is

$$\int^t x_n(dt')dt'$$

If the message signal is a sinusoid, the integrator output is a sinusoid having the same frequency; but the (input and output) amplitudes are different and the phases are different.

The Armstrong phase modulator looks similar to a modulator for AM. For both modulators, a double-sideband term and a residual carrier are added together. Furthermore, the spectrum of the output from the Armstrong modulator looks similar to the spectrum of AM. For a sinusoidal message signal, there are three spectral lines: one from the residual carrier and two from the double-sideband term. However, an Armstrong modulator does not produce an AM carrier. In the Armstrong modulator the phase shifter (which delays the residual carrier by $\pi/2$ radians) makes the difference.

In the remainder of this section, the assumption is made that $x_n(t)$ is $\cos(2\pi f_m t)$. Noting that $\cos(2\pi f_c t)$ with a delay of $\pi/2$ radians is $\sin(2\pi f_c t)$, the output of the Armstrong modulator is of the form

$$a \sin(2\pi f_c t) + b \sin(2\pi f_m t) \cos(2\pi f_c t) = \sqrt{a^2 + b^2 \sin^2(2\pi f_m t)} \cdot \sin[2\pi f_c t + \tan^{-1}(b \sin(2\pi f_m t)/a)] \quad (20)$$

This signal shows both amplitude and angle modulation.

If $|b|$ is small compared with $|a|$, then Eq. (20) can be approximated as

$$\sqrt{a^2 + b^2 \sin^2(2\pi f_m t)} \cdot \sin[2\pi f_c t + \tan^{-1}(b \sin(2\pi f_m t)/a)] \cong |a| \sin[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad (21)$$

where

$$\beta = \tan^{-1} \left(\frac{b}{a} \right) \quad (22)$$

β is the modulation index discussed previously. A comparison of Eqs. (12) and (21) shows that the right-hand side of Eq. (21) has the form of FM with a sinusoidal message signal.

One method of setting the modulation index β is to individually set the parameters a and b such that they are related to the desired value of β through Eq. (22). In the laboratory it is generally more convenient to measure the rms value of signals, rather than the peak values ($|a|$ and $|b|$). The relationship between rms and peak values depends on the type of signal. The table below shows these relationships for the signals of interest here.

<u>Signal</u>	<u>Example</u>	<u>rms</u>
sinusoid	$a \sin(2\pi f_c t)$	$ a /\sqrt{2}$
product of sinusoids	$b \sin(2\pi f_m t) \cos(2\pi f_c t)$	$ b /2$

The following rms voltages can be measured:

V_{DSB}	rms voltage of double-sideband (DSB) component at modulator output
V_{C}	rms voltage of residual carrier (C) component at modulator output

V_{DSB} can be measured by temporarily disconnecting the residual carrier from the modulator. Similarly, V_{C} can be measured by temporarily disconnecting the DSB component from the modulator (so that only the residual carrier is present). The modulation index can then be calculated as

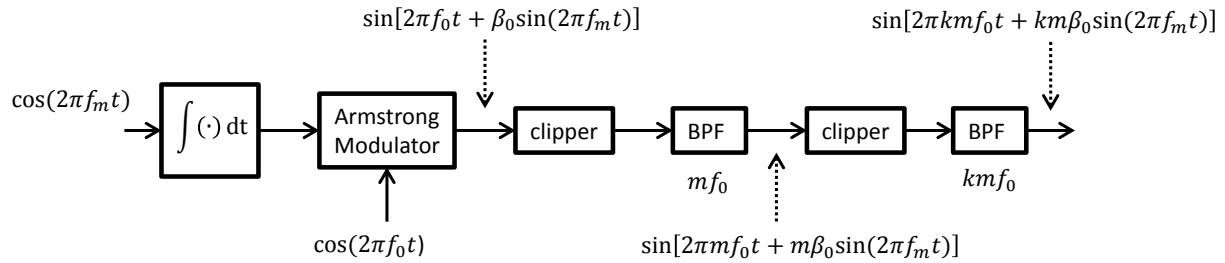
$$\beta = \tan^{-1} \left(\frac{\sqrt{2} V_{\text{DSB}}}{V_{\text{C}}} \right) \quad (23)$$

The generation of FM by the indirect method, employing an integrator and phase modulator, has an important practical advantage over the direct method. A VCO typically has poor frequency stability, so a carrier generated from a VCO (direct method) exhibits frequency drift. With the

indirect method a very stable oscillator (usually a crystal oscillator) can be used as the source of the carrier.

0.6 Frequency Multiplication

An integrator followed by an Armstrong phase modulator can produce a good approximation of FM when the desired modulation index β is small. This is called *narrowband FM*, owing to the fact that the modulation index and therefore the bandwidth is small. However, the well-known performance advantage of FM over AM occurs only when β is large. However, there is a way to use an Armstrong modulator and yet still achieve a large β . This is illustrated below. (For the sake of having a definite example, the illustration below assumes the message signal is a sinusoid.)



In words, the FM is generated with an integrator followed by an Armstrong phase modulator. The carrier frequency f_0 and the modulation index β_0 at the output of the Armstrong modulator are both smaller than desired. This narrowband FM signal is placed at the input of a (memoryless) nonlinear device (such as a clipper), which generates harmonics. Then the m -th harmonic is selected by a bandpass filter, having a passband centered at mf_0 . The output of this bandpass filter is a carrier of frequency mf_0 , having FM with modulation index $m\beta_0$. The combination of the nonlinear device and the bandpass filter is called a *frequency multiplier*. The truth be told, *angle multiplier* would be a better name, because it is the angle (the argument of the carrier sinusoid) that is multiplied. A second frequency multiplier can be used to further increase both the modulation index and the carrier frequency. If the second frequency multiplier employs a factor k (selects the k -th harmonic), then the output of this second frequency multiplier is a carrier of frequency kmf_0 , having FM with modulation index $km\beta_0$. The following equations summarize the effect of the frequency multiplications:

$$f_c = kmf_0 \quad (24)$$

$$\beta = km\beta_0 \quad (25)$$

In this way, quite large modulation indices can be achieved. Because this is the indirect method for generating FM, which permits use of a crystal oscillator for the frequency f_0 , the carrier can have excellent frequency stability.

0.7 Bessel Null Method

The *Bessel null method* (also called the *carrier null method*) is an experimental technique used in FM systems for setting the frequency deviation Δf with excellent precision. This technique is based on the following observation. When the message signal is a sinusoid, the modulated carrier has a spectrum consisting of spectral lines. The carrier line (that is, the residual carrier) disappears when β has a value corresponding to a *null* for the Bessel function $J_0(\beta)$. Table 2 identifies β for the first two carrier nulls.

Table 2: Some Carrier Nulls

Carrier Null	β
first	2.405
second	5.520

This method uses a sinusoidal message signal for calibration, but an FM system that employs a general (that is, non-sinusoidal) message signal can benefit from this calibration. Here is the procedure:

1. Calculate the frequency f_m such that $f_m = \Delta f / \beta$, where Δf is the desired frequency deviation and β is a value from Table 2.
2. Adjust the frequency of a test oscillator to f_m .
3. Use the test oscillator as the message signal, observe the spectrum of the modulated carrier, and adjust the sensitivity of the frequency modulator for the selected carrier null.
4. Replace the test oscillator with the genuine message signal.

It is essential that the peak amplitude of the genuine message signal be the same as the peak amplitude of the test sinusoid. Any carrier null can be used in this procedure; however, it is necessary that the value for β used in Step 1 correspond to the same carrier null as observed in Step 3. For example, $\beta = 2.405$ can be used in Step 1, then the modulator sensitivity can be adjusted for the first carrier null in Step 3.

1 FM with VCO

You will generate FM using a VCO. This is the direct method. You will also demonstrate that a carrier with FM can be detected with a zero-crossing detector.

1.1 Direct Method of Generating FM

Make sure the switch on the VCO module's PCB is set to "VCO". Set the toggle switch on the front panel of the VCO module to "HI". Place the VCO output on the input of the Frequency Counter. With the gain knob on the VCO module set in the fully counterclockwise (zero-gain) position, use the frequency knob to set the VCO output frequency to approximately 100 kHz.

You will probably see the frequency slowly drift because the VCO does not have good frequency stability. (This is unlike the signals coming from the Master Signals module. These are synthesized from a crystal oscillator.)

In the first instance, connect the Variable DC output to the VCO input. Adjust the DC source for a positive voltage, confirming this with the oscilloscope. Rotate the gain knob on the VCO clockwise until the VCO output frequency changes significantly from the original 100 kHz. K_{VCO} for this VCO is negative, so the VCO output frequency should decrease in response to the positive input voltage.

For this experiment it is desirable to have the VCO frequency increase in response to a positive voltage. Therefore, the Variable DC output should be disconnected from the VCO and then connected through a Buffer Amplifier (having negative gain) to the VCO. You should now observe that a positive voltage on the Variable DC output produces an increase in the VCO frequency.

Adjust the Variable DC output to +2 V. Then adjust the gain of the Buffer Amplifier so that its output is -1 V. The RMS Meter can provide excellent accuracy in measuring the absolute value of these DC voltages; however, the oscilloscope is needed in order to verify the correct algebraic sign on these voltages.

Now adjust the gain on the VCO so that the VCO output frequency is approximately 1 kHz larger than its value with no input voltage. The sensitivity K_{VCO} now has the approximate value -1 kHz/V. Furthermore, the product GK_{VCO} currently has the value +500 Hz/V. In the following experimental procedure, the gain knob on the VCO will remain untouched but the Buffer Amplifier gain will be changed. K_{VCO} will retain the value -1 kHz/V. G , and therefore the product GK_{VCO} , will change.

Complete the following table. The first column is the DC voltage at the VCO input (not the Variable DC output). The second column is the measured VCO output frequency minus the VCO frequency in the absence of an input. Because K_{VCO} is negative, the frequency offset should be positive when the VCO input voltage is negative and vice versa. It will be necessary to adjust both the Variable DC output and the Buffer Amplifier gain in order to reach all voltages in the range -8 V to +8 V. If you plot the data in this table, this should approximate a straight line with slope -1 kHz/V.

DC voltage at VCO input	Frequency offset (kHz)
-8	
-6	
-4	
-2	
+0	
+2	
+4	
+6	
+8	

Replace the DC voltage from the Variable DC module with a sinusoid from the Audio Oscillator. The output of the Audio Oscillator should pass through the Buffer Amplifier to the VCO input. Adjust the frequency of the Audio Oscillator to approximately 1 kHz. Monitor the rms voltage at the VCO input. Adjust the gain of the Buffer Amplifier so that the VCO input is a sinusoid with a peak voltage of 7 V. (What is the corresponding rms voltage for this sinusoid?) Observe the VCO output on the oscilloscope. Use the VCO output as the trigger. The display won't be stable because the signal depends on two different, unrelated frequencies: the nominal carrier frequency (set in the VCO) and the 1-kHz sinusoid (from the Audio Oscillator). Nonetheless, the display should suggest that the instantaneous frequency of the VCO output is changing in a periodic way.

Add a frequency measurement to the oscilloscope display. This can be done with the following selection from the PicoScope *Measurements* menu:

Measurements > Add Measurement

A dialog box will pop up, and you should select a frequency measurement on the appropriate channel. The result of this selection will be a measurement table in the PicoScope display. The average, maximum and minimum frequencies will be indicated in the table. Compute and record the following:

$$\begin{aligned} & \text{Maximum Frequency} - \text{Average Frequency} \\ & \text{Average Frequency} - \text{Minimum Frequency} \end{aligned}$$

Each of these frequency differences should equal approximately the frequency deviation as given by Eq. (7). (You should know the product pG and the VCO sensitivity K_{VCO} .)

Readjust the Audio Oscillator frequency to 5 kHz. When changing the frequency of the Audio Oscillator, the amplitude of the sinusoid may also change a little. Readjust the gain of the Buffer Amplifier so that the VCO input again has a peak voltage of 7 V. Observe how the maximum

and minimum values of the instantaneous frequency are approximately the same as before. The frequency deviation is independent of the frequency of the message signal.

1.2 Spectrum of FM

The VCO sensitivity K_{VCO} should be kept the same as above (approximately -1 kHz/V). Adjust the Audio Oscillator for a frequency of approximately 1 kHz. The Audio Oscillator output should pass through the Buffer Amplifier to the VCO input (as above). The VCO output should be observed on the PicoScope (Spectrum Mode). Complete the following table.

Δf (kHz)	β	Carson Bandwidth (kHz)	Residual Carrier (dBu)	Upper Fundamental (dBu)	P_1/P_0 (dBc)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					

For each row in the above table, you will set the frequency deviation with the aid of Eq. (7) by adjusting the product pG to the correct value. To help you do this, you should monitor the VCO input with the RMS Meter and take into account the relationship between an rms voltage and the corresponding peak voltage (such as pG). The second and third columns can be completed using Eqs. (11) and (19), respectively.

For each row in the table above, you should compare the spectrum with that expected from theory. You should consider whether the Carson's rule estimate of FM bandwidth, Eq. (19), is reasonable based on the observed spectrum. It will be helpful to zoom in to that part of the spectrum near the residual carrier. You can do this by selecting zoom and then changing the position and size of the Zoom Overview box.

The fourth and fifth columns in the above table will contain measurement values. The fourth column is the line height of the residual carrier. The fifth column is the line height of the upper fundamental. (The line height of the lower fundamental should be approximately the same as that for the upper fundamental.) A convenient and accurate method for making these measurements is to use two vertical rulers. Place one vertical ruler approximately at the frequency of the residual carrier and the second at approximately the frequency of the upper fundamental. Add two measurements to the spectrum display.

Measurements > Add Measurement

A dialog box will pop up, and you should select an *amplitude at peak* measurement for the *peak nearest ruler 1*. Repeat this procedure for a second amplitude measurement for the *peak nearest ruler 2*.

The final column of the above table will contain the upper fundamental power in dBu (column 5) minus the residual carrier power in dBu (column 4). You should compare your measurement results with the predictions (based on theory) of Table 1.

1.3 Zero-Crossing Detector

A carrier with FM will now be detected using a zero-crossing detector. First, generate a carrier with FM. You can use the same set-up as above: the Audio Oscillator output passes through a Buffer Amplifier to the VCO input.

Set the toggle switch on the front panel of the VCO module to “LO” and then adjust the nominal frequency of the VCO to 10 kHz. For this demonstration, the residual carrier will have an unusually small frequency because this provides a clearer view of the detection process on the oscilloscope.

Set the VCO sensitivity K_{VCO} to approximately -1 kHz/V. Even though you had earlier adjusted the VCO gain knob to this sensitivity, you should check this. Changing the VCO nominal frequency to 10 kHz might have affected the sensitivity.

Adjust the Audio Oscillator frequency to approximately 500 Hz. Adjust the gain of the Buffer Amplifier so that the peak voltage at the VCO input is approximately 7 V.

The Twin Pulse Generator will be part of the zero-crossing detector. Make sure the slide switch on the PCB of this module is set to *single* mode. Rotate the width knob so that the line on the knob points vertically, so that the pulse width is not too narrow and not too wide.

Connect the FM carrier (the output of the VCO) to the analog signal input of the comparator on the Utilities module. The analog reference input to the comparator should be ground. Connect the comparator TTL output to the clock input of the Twin Pulse Generator. The combination of the comparator and the Twin Pulse Generator constitutes a zero-crossing detector. Connect the Q_1 output of the Twin Pulse Generator to the input of a Tuneable LPF. Adjust the filter bandwidth to approximately 1 kHz.

Simultaneously observe the VCO output and the zero-crossing detector output (the Q_1 output of the Twin Pulse Generator) on the oscilloscope. Even though it won't be possible to get a stable display (without stopping signal capture), for best viewing you should use the VCO output as the trigger source. You can occasionally stop data capture in order to get snapshots of the signals. This oscilloscope display demonstrates the action of the zero-crossing detector.

Simultaneously observe the zero-crossing detector output and the LPF output on the oscilloscope. Use the TTL output from the Audio Oscillator as an external trigger source. (As always with a TTL signal, the trigger level must be positive.) This display demonstrates the action of the LPF on the pulsed signal coming from the zero-crossing detector.

Simultaneously observe the original message signal (the Audio Oscillator output) and the LPF output on the oscilloscope. Use the TTL output from the Audio Oscillator as an external trigger source. The LPF output should be a reasonable approximation of the original message signal. There will be delay, and the amplitude will be different. However, the LPF output should approximate a sinusoid of the correct frequency.

1.4 FM with Speech

Change the VCO nominal frequency back to 100 kHz by setting the toggle switch back to “HI” and adjusting the frequency knob. Recalibrate the VCO sensitivity K_{VCO} for -1 kHz/V.

Replace the Audio Oscillator with a speech signal.

Other than changing the message signal and changing the carrier frequency back to 100 kHz, the configuration will be the same as for the zero-crossing demonstration. Connect the output of the LPF to the headphones and listen to the demodulated signal. Adjust the gain of the Buffer Amplifier in the modulator and the bandwidth of the LPF in the demodulator as necessary to produce good sound quality at the output of the demodulator.

2 Armstrong Modulator and Frequency Multiplication

In this part of the experiment, you will build an Armstrong (phase) modulator. With an integrator placed before the Armstrong modulator, the combination becomes an indirect method for generating FM. You will begin by experimenting with the integrator. In the first Armstrong modulator that you build, the carrier frequency will be 100 kHz.

The combination of integrator and Armstrong modulator produces a good approximation to FM only for small modulation indices. You will demonstrate that this limitation can be overcome by following the Armstrong modulator with frequency multipliers. These devices multiply the modulation index as well as the carrier frequency. In this experiment, you will use a cascade of two frequency multipliers, each with multiplication factor 3. Therefore, the modulation index at the output of the entire chain will be 9 times the modulation index produced at the immediate output of the Armstrong modulator. The carrier frequency also gets multiplied by 9, so the carrier used within the Armstrong modulator must be $(100/9)$ kHz if the carrier at the output of the entire chain is to be 100 kHz.

2.1 Integrator

You will use the s^{-1} function in the Laplace Transform module to perform the integration required before the Armstrong (phase) modulator in order to achieve a frequency modulator. A linear time-invariant system whose transfer function is s^{-1} is an indefinite integrator. However, the so-called s^{-1} function in the Laplace Transform module actually produces

$$C - \int x(t') dt'$$

at its output, where C is a constant and $x(t)$ is the analog input. The output of the Laplace module will go to a multiplier that will be set for AC coupling, and that will eliminate the unwanted DC component C . The negative sign in front of the indefinite integral is not desirable; but, on the other hand, it won't be a real problem. The message signal $x(t)$ will be a sinusoid here, so the negative sign just shifts the phase of the message sinusoid by 180° .

The indefinite integral of a sinusoid is a sinusoid of the same frequency but with a different amplitude (which is inversely proportional to the frequency) and a different phase. The phase change is $-\pi/2$ radians for an indefinite integral. For an indefinite integral with a negative sign in front, the phase change is $\pi - \pi/2 = +\pi/2$ radians. That is to say, the output sinusoid will lead the input by 90° .

Place a sinusoid from the Audio Oscillator on the input of the (Laplace Transform module's) s^{-1} function and also on Channel A of the oscilloscope. Place the output on Channel B. Since this integrator has a negative sign, the output should lead the input by 90° . Try a few different frequencies for the sinusoid produced by the Audio Oscillator. You should observe the expected phase relationship between the Channel A and B sinusoids. Switch the oscilloscope to X-Y View

Views > X-Axis > A

You will observe an ellipse. If the phase difference between the two sinusoids is truly 90° , the one axis of the ellipse will be horizontal and the other vertical. As you change the frequency of the input sinusoid, you should observe that the relative lengths of the axes of the ellipse change, since the amplitude of the integrator output is inversely proportional to the frequency. But the axes should remain horizontal and vertical.

2.2 Armstrong Modulator

Construct an Armstrong (phase) modulator. Place the output of the (Laplace Transform module's) s^{-1} function at the input of the Armstrong modulator. Set the Multiplier within the Armstrong modulator for AC coupling, in order to block the (unwanted) DC component on the

arriving signal. The output of the Armstrong modulator (with preceding integrator) will approximate FM when the modulation index is small.

For now use a 100-kHz sinusoid (Master Signals) as the unmodulated carrier for the Armstrong modulator. (This will change later.) Set the frequency-range switch on the Phase Shifter PCB to “HI”. Adjust the delay of the Phase Shifter so that it changes the phase of a 100-kHz sinusoid by $-\pi/2$ radians.

Include a (negative-gain) Buffer Amplifier after the (weighted) Adder, in order to cancel the (negative) weighting factors of the Adder.

The 2-kHz analog sinusoid (Master Signals) will serve as the message signal. Therefore, it should be placed at the input to the integrator.

Adjust the modulation index β to 0.5. You can accomplish this with the help of Eq. (23), recognizing that V_C is the rms voltage at the modulator output with only the residual-carrier component present and V_{DSB} is the rms voltage with only the DSB component present.

Display the modulator output on the oscilloscope, using the modulator output as the trigger source. You should observe that both frequency and amplitude modulation are present. With this small modulation index, however, the amplitude modulation should be relatively small.

Now observe the spectrum of the modulator output. Measure P_1/P_0 . Compare this measurement with what might be expected, based on Table 1.

2.3 Frequency Multipliers

You will build two frequency multipliers. For this purpose you will need the 33.3-kHz bandpass filter on the FM Utilities module and the 100-kHz bandpass filter on the 100-kHz Channel Filters module, with the module switch in Position 3.

Obtain a quick view of the magnitude of the transfer function for each of these filters. You can do this by connecting a (100/96)-kHz TTL clock to the clock input of the Twin Pulse Generator and connecting the Q_1 output of this generator to the filter input. Make sure the Twin Pulse Generator is set for *single* mode and the pulse width is adjusted to its minimum value. (This creates an approximation of a Dirac comb.) With this input to each filter, the spectrum of the filter output provides an outline of the magnitude of the transfer function.

Use the FM Utilities module to create a stable 11.1-kHz sinusoid. You can do this by providing a 100-kHz sinusoid (Master Signals) to the input port that expects a 100-kHz sinusoid. The adjacent output port then supplies a sinusoid with a frequency of (100/9) kHz, approximately 11.1 kHz.

In the Armstrong modulator, replace the 100-kHz unmodulated carrier with the 11.1-kHz sinusoid. It is essential that you readjust the delay of the Phase Shifter so that a 11.1-kHz sinusoid experiences a phase change of $-\pi/2$ radians. (Recall that the Phase Shifter introduces a phase delay that depends on the frequency.)

The integrator should remain in place (providing the input to the Armstrong phase modulator). The input to the integrator should be the 2-kHz analog sinusoid (Master Signals).

The output of this Armstrong modulator is now an 11.1-kHz carrier with FM. Connect the output of the Armstrong modulator to a sequence of two frequency multipliers, each multiplying the frequency and modulation index by 3. The output of the second frequency multiplier will therefore be a carrier of frequency 100 kHz with a modulation index that is 9 times that of the Armstrong modulator. Each frequency multiplier will consist of a clipper (a nonlinear device that creates odd harmonics) followed by a bandpass filter. The two clippers are available on the FM Utilities module. The 33.3-kHz bandpass filter is used in the first frequency multiplier, and the 100-kHz bandpass filter is used in the second frequency multiplier.

Observe the output of the second frequency multiplier on the oscilloscope, using this signal as its own trigger. You should observe frequency modulation.

The modulation index at the output of the Armstrong modulator shall now be called β_1 , and the modulation index at the output of the second frequency multiplier shall be called β_9 . Therefore,

$$\beta_9 = 9\beta_1 \quad (26)$$

Complete the following table. For each row, there is a desired modulation index β_9 at the output of the second frequency multiplier. For each β_9 , you should perform the following tasks. First, compute the corresponding value of β_1 with the help of Eq. (26). Second, implement this modulation index in the Armstrong modulator with the help of Eq. (23). Third, observe (sequentially) the spectrum at the output of the Armstrong modulator, at the output of the first frequency multiplier, and at the output of the second frequency multiplier. (It should be clear from the spectra that the modulation index increases with each frequency multiplication.) Fourth, measure, at the output of the second frequency multiplier, the line height of the residual carrier and of the upper fundamental. Fifth, compute P_1/P_0 based on your measurements. Sixth, compare the measured P_1/P_0 with the expected value from Table 1.

β_9	Residual Carrier (dBu)	Upper Fundamental (dBu)	P_1/P_0 (dBc)
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			

2.4 Bessel Null Method

Here you will use the direct method of generating FM (see Subsection 1.1). Connect the analog output of the Audio Oscillator to the input of a Buffer Amplifier and the output of the Buffer Amplifier to the VCO input.

Use the Bessel null method to adjust the frequency deviation Δf to 2.0 kHz. This will require you to complete Steps 1, 2 and 3 of the procedure given in Subsection 0.7. In Step 1, use a β of 2.405, corresponding to the first carrier null. In Step 3, when you are to observe the spectrum of the modulated carrier, you should adjust the VCO sensitivity until the first carrier null is achieved. Also, using the RMS Meter, make a note of the rms voltage at the VCO input.

You should now assume that the genuine message signal is a 1-kHz sinusoid. The Audio Oscillator will supply this genuine message signal. Adjust its frequency to 1.0 kHz. In order that Δf not change from the value you just set, two precautions must be taken. First, the VCO sensitivity must not change. Second, the peak amplitude of the genuine message signal must match that of the test sinusoid (which had a frequency of $\Delta f/\beta$). When you change the frequency of the Audio Oscillator, its amplitude may also change somewhat. Therefore, you must monitor the rms voltage at the VCO input and adjust the gain of the Buffer Amplifier (that connects the Audio Oscillator to the VCO input) until the rms voltage at the VCO input is the same as before (when the frequency was $\Delta f/\beta$).

If you have followed the procedure correctly, the frequency deviation Δf for the modulated carrier should still be 2.0 kHz. The modulation index β , on the other hand, will have changed. Calculate the modulation index with the message signal of 1.0 kHz. Observe the spectrum of the modulated carrier and measure P_1/P_0 . You should compare this with what you would expect based on Table 1.